647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov - 2025 (NEW COURSE)
Boolean Algebra & Complex Analysis -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two: (10)

(1) Define: Equivalence relation. Show that similarity of matrices on the set of $n \times n$ matrices is an equivalence relation.

(2) If $(L, \leq)$ is a lattice and $a, b, c \in L$ then prove that :

(i) $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

(ii) $b \leq c \Longrightarrow a * b \leq a * c$ and $a \oplus b \leq a \oplus c$

(3) Define: Direct product of two lattice. Prove that the direct product of two lattice is also a lattice.

Que.1(B) Answer any one: (04)

(1) Define: cover of an element of a poset. Draw the Hasse diagram of (S_{12}, D).

(2) Define: Partially ordered set and totally ordered set. Give an example of Partially ordered set which is not totally ordered set.

Que.2(A) Answer any two: (10)

(1) State and prove D'Morgan's laws for Boolean algebra.

(2) Define: Karnaugh Map (K-Map) of two variables. Obtain the minimal sum of the product of the Boolean expression $\alpha(x, y) = xy + xy'$ by Karnaugh map.

(3) State and prove the unique representation theorem for Boolean algebra.

Que.2(B) Answer any one: (04)

(1) If a and b are distinct atoms of the Boolean algebra $(B, *, \oplus, ', 0, 1)$ then prove that $a * b = 0$.

(2) Express the Boolean expression $\alpha(x_1, x_2, x_3) = x_2 * x_3$ as the product of sums canonical form.

Que.3(A) Answer any two: (10)

(1) If complex function $f(z) = u + iv$ is analytic at $z_0 = x_0 + iy_0$ then prove that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

(2) Obtain Laplace equation in polar form.

(3) Prove that $u = r^2 sin2\theta$ is harmonic function and find its conjugate.

Que.3(B) Answer any one: (04)

(1) Define: Entire function. Show that $f(z) = (3x + y) + i(3y - x)$ is an entire function.

(2) If u and v are conjugate harmonic functions then prove that the family of curves obtained by $u = c_1$ and $v = c_2$ are orthogonal.

Que.4(A) Answer any two: (10)

(1) In usual notation, prove that: $\int_c f(z)dz = ir_0 \int_0^{2\pi} f\left(z_0 + re^{-i\theta}\right) e^{i\theta} d\theta$, where $c: z - z_0 = r_0 e^{i\theta}$ and hence deduce that: $\int_c \frac{dz}{z - z_0} = 2\pi i$.

(2) In usual notations prove that: $\left|\int_a^b f(z)dz\right| \leq \int_a^b |f(z)| dz$.

(3) Define: Closed curve, prove that $\int_c \pi e^{\pi \bar{z}} dz = 4(e^\pi - 1)$, where C is the square joining points $z = 0, z = 1, z = 1 + i$ and $z = i$.

Que.4(B) Answer any one: (04)

(1) Find: $\int_{i}^{1+i}(x-y+ix^2)dz$

(2) Find: $\int_{c} \frac{z}{(2z-1)(z+1)}dz,\ c:|z|=2.$

Que.5(A) Answer any two: (10)

(1) State and prove Morera's theorem.

(2) Prove that: $\int_{c} \frac{e^{kz}}{z}dz = 2\pi i,\ c:|z|=1$ and hence deduce that,

(i) $\int_{0}^{\pi} e^{kcos\theta}cos(ksin\theta)d\theta = \pi$

(ii) $\int_{-\pi}^{\pi} e^{kcos\theta}sin(ksin\theta)d\theta = 0$

(3) State Cauchy's inequality and prove that if f is entire and bounded for all values of z in the complex plane then $f(z)$ is constant through the plane.

Que.5(B) Answer any one: (04)

(1) If $g(z_0) = \int_{c} \frac{z^2+2z+5}{z-z_0}dz, c:|z|=3$ then find $g(0), g(1), g'(-2), g(i)$ and $g(z_0)$ when $|z_0| > 3$.

(2) Evaluate: $\int_{c} \frac{z^2+3}{z^2(z-4)}dz,\ c:|z|=1.$

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